Impedance Spectrum of Batteries Determined by Wavelet Transformation -Discussion on Optimum Conditions for Wavelet Transformation-

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Electrochemical impedance spectroscopy (EIS) is useful method to analyze the electrode structure and condition because time constants in the electrochemical response can be discriminated by EIS. The impedance spectrum is usually measured by using a frequency response analyzer (FRA) whose principle is on the basis of Fourier transformation (FT). FT is the calculation method to transform data in the time domain to those in the frequency domain. On the other hand, Itagaki *et al* [1,2,3)] applied Wavelet Transform (WT) to EIS and proposed the calculation method of impedance spectrum from time series data of current and voltage signals by using WT.

WT is one of the frequency analysis methods. WT can transform data in the time domain to those in the frequency domain retaining time information. The wavelet coefficient $\tilde{x}$ of a signal $x(t)$ is calculated by the convolution integral of $x(t)$ and the function Ψ called "mother wavelet".

$$\tilde{x}(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\,\psi\left(\frac{t-b}{a}\right) dt \qquad (1)$$

In equation (1), a is the scale parameter, b is the time parameter. Impedance spectrum can be calculated by WT with the complex Morlet wavelet for mother wavelet which multiplied the sinusoidal wave by the gauss function as a window as follows.

$$\psi\left(\frac{t-b}{a}\right) = \frac{1}{\sqrt{\pi\sigma}} exp \frac{j2\pi f_c (t-b)}{a} exp\left(-\frac{(t-b)^2}{a^2\sigma}\right) \qquad (2)$$

In equation (2), f_c is the center frequency [Hz] and σ is bandwidth parameter [s^2]. The analysis frequency (f) is represented as follows .

$$f = \frac{f_c}{a} \qquad (3)$$

This mother wavelet is represented by the product of sinusoidal term $\exp(j2\pi f_c t_m)$ and Gaussian function. The waveform of complex Morlet mother wavelet is shown in Fig. 1. The wave is composed of real (r) and imaginary (i) components. The length of time window of $\Psi(t)$ can be adjusted by the value of σ.

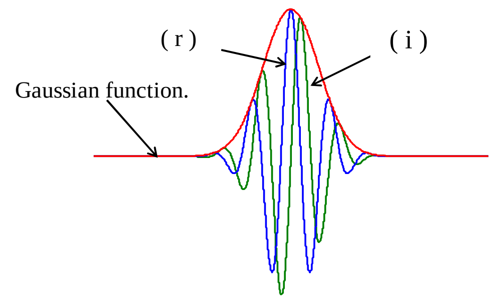

Fig. 1 The schematic diagram of complex Morlet mother wavelet

In the present work, the conditions of following parameters on the impedance spectra were discussed.

(1) Suitable sampling time to calculate right impedance spectrum
(2) Suitable ratio of f_c and σ involved in the equations (2) and (3)
(3) Necessary signal length to analyze impedance at the low frequency limit

In order to examine the above-mentioned theory, the measurement of the output signals of current and voltage was performed by turning on and off of the secondary battery. The each signal is transformed from time domain to frequency domain using WT. The electrochemical impedance $Z(a,b)$, whose time and frequency is represented by b and f_c/a respectively, can be obtained by wavelet coefficients of voltage $\tilde{f}(a,b)_{\rm vol}$ and current $\tilde{f}(a,b)_{\rm cur}$ signals as follows.

$$Z(a,b) = \frac{\tilde{f}(a,b)_{\rm vol}}{\tilde{f}(a,b)_{\rm cur}} \qquad (4)$$

The details of the results of output signals of current and voltage and the calculation result of impedance spectrum by using WT will be discussed.

References

1) M. Itagaki, T. Saito, I. Shitanda, *221th ECS Meeting abstract* #1585 (2012).
2) M. Itagaki, T. Saito, I. Shitanda, *220th ECS Meeting abstract* #2628 (2011).
3) M. Itagaki, T. Saito, K.Isobe, I. Shitanda, Proceedings of the 6th Japan-China Joint Seminar on Marine Corrosion ,1(2012).