Tutorial: The Power Law Model for Interpretation of CPE Parameters

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Jorcin et al.[1] have suggested that Constant-Phase Element (CPE) behavior can be attributed to surface and/or normal distributions of time constants. This powerful insight has facilitated categorization of different models for extracting meaningful physical properties from CPE parameters. While the model proposed by Brug et al.[2] seems best suited for surface distributions, the power-law model has recently emerged as a powerful tool for interpretation of constant-phase element (CPE) parameters in terms of the properties of dielectric films.

Models invoking Constant-Phase Elements (CPE) are often used to fit impedance data arising from a broad range of experimental systems. While the physical origins of the CPE are controversial, a bigger problem remains the interpretation of impedance data in terms of physically meaningful properties such as capacitance or thickness. Often, the formula attributed to Brug et al.,[2]

$$C_{\text{eff}} = \frac{\epsilon\epsilon_0}{\delta} = Q^{1/\alpha}\left(\frac{R_\text{e}+R_\text{t}}{R_\text{e}R_\text{t}}\right)^{(\alpha-1)/\alpha} \qquad (1)$$

is invoked, because the parameter values obtained seem reasonable. This formula was derived, however, for a surface distribution of capacitance and does not apply to the normal distribution of time constants expected for a dielectric response.[3]

A formula attributed to Hsu and Mansfeld,[4]

$$C_{\text{eff}} = \frac{\epsilon\epsilon_0}{\delta} = Q^{1/\alpha} R_\text{p}^{(1-\alpha)/\alpha} \qquad (2)$$

can be associated with a normal distribution of time constants,[3] but the physical properties obtained are often physically unreasonable.

Hirschorn et al.[5] developed an alternative approach which was applied to various experimental systems.[6,7] By assuming that the normal distribution of time constants could be attributed to a distribution of resistivity with a uniform dielectric constant, they found

$$C_{\text{eff}} = \frac{\epsilon\epsilon_0}{\delta} = Q(\rho_\delta \epsilon\epsilon_0)^{1-\alpha} g \qquad (3)$$

where ρ_δ is the lower limit of the resistivity evaluated at $x=\delta$ and g is a known function of α, varying between 1 and 1.6 for $0.5<\alpha<1$. In a subsequent paper, Musiani et al.[8] demonstrated that, for systems showing CPE behavior, a distribution of dielectric constant does not change the essential behavior, and Eqn. (3) still applies, albeit with ϵ evaluated at $x=\delta$, where the local resistivity has its smallest value.

Equations (1) and (2) have the attractive feature that, as all parameters may be known, they provide an unambiguous values for film thickness or dielectric constant. Recent work has shown that, while the values provided by equations (1) and (2) are unambiguous, they are incorrect.[9] Equation (3) was shown to provide correct values, but, as the value for the parameter ρ_δ is usually uncertain, the values for dielectric constant or film thickness are correspondingly uncertain.

This paper will provide a review of the methods used to assess film properties from CPE parameters, with emphasis on the power-law model. Error propagation analyses will be used to show the level of certainty that can be applied to parameters obtained from the power-law model. This work shows that physically meaningful parameters may be obtained from CPE parameters.

References

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